$$\lambda_P = \frac{l_1}{l_0} = \frac{\sin \epsilon_0}{\sin \epsilon_1} \qquad [40]$$

Eq. [40], which relates the tensile elongation with the lattice rotation for a crystal undergoing double glide, was obtained by v. Göler and Sachs[4] through the integration of a differential equation. From Eq. [6], we also have

$$\lambda_P p_3 = \frac{\partial x_3}{\partial X_1} P_1 + \frac{\partial x_3}{\partial X_2} P_2 + \frac{\partial x_3}{\partial X_3} P_3$$

or

$$\lambda_P \cos \epsilon_1 = -\frac{1}{\sqrt{2}} (1 - e^{\varphi}) \sin \epsilon_0 + e^{\varphi} \cos \epsilon_0$$

or

$$e^{\varphi} = \frac{\sqrt{2} \cot \epsilon_1 + 1}{\sqrt{2} \cot \epsilon_0 + 1} \qquad [41]$$

by substituting $\lambda_P = \sin \epsilon_0 / \sin \epsilon_1$. Eq. [41] may be rewritten as

$$S = 2\alpha = \sqrt{6}\varphi = \sqrt{6} \ln \left[\frac{\sqrt{2} \cot \epsilon_1 + 1}{\sqrt{2} \cot \epsilon_0 + 1}\right] \qquad [42]$$

which relates the amount of glide and the lattice rotation for the double-glide case. Eq. [42] was likewise developed by v. Göler and Sachs. It may be noted if ϵ_0 and ϵ_1 are measured from $[\bar{1}12]$ and toward the $[\bar{1}11]$ position, Eq. [42] becomes

$$S = \sqrt{6} \ln \left[\frac{\sqrt{2} \cot \epsilon_1 - 1}{\sqrt{2} \cot \epsilon_0 - 1}\right] \qquad [43]$$

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*Note added in proof: After the present work was submitted for publication, two related papers, by Bowen and Christian[10] and by Schubert,[11] have come to our attention. The Bowen and Christian treatment of single glide is essentially the same as ours. Their results for double glide, like those of v. Göler and Sachs, were obtained by integrating a differential equation. The latter method was also used by Schubert in treating both single and double glide. On the other hand, we obtained, directly from the limit in Eq.[21], the resultant deformation gradient matrix, from which all quantities associated with the deformation can be computed readily.

APPENDIX[9]

ALTERNATIVE EVALUATION OF $e^{\alpha F_1}$ IN THE DOUBLE-GLIDE CASE

In Eq. [20], it is noted that

$$F_1 = m_A n_A^T + \beta m_B n_B^T \qquad [A.1]$$

for slip on systems A and B. Let us define matrices

$$P = m_A n_A^T, \quad Q = m_B n_B^T, \quad R = m_B n_A^T, \quad S = m_A n_B^T \qquad [A.2]$$

and the scalar products

$$r = n_A^T m_B, \qquad s = n_B^T m_A \qquad [A.3]$$

Since the slip directions $\mathbf{m}_A$ and $\mathbf{m}_B$ lie in the slip planes of normals $\mathbf{n}_A$ and $\mathbf{n}_B$, respectively, we have

$$n_A^T m_A = n_B^T m_B = 0 \qquad [A.4]$$

In view of the above definitions, the matrices $P, Q, R,$ and S have the following multiplication table:

[A.5]

First Factor \ Second Factor	P	Q	R	S
P	0	rS	rP	0
Q	sR	0	0	sQ
R	0	rQ	rR	0
S	sP	0	0	sS

By application of this table, one finds

$$F_1^2 = (P + \beta Q)^2 = P^2 + \beta(QP + PQ + \beta Q^2) = \beta(sR + rS)$$

$$F_1^3 = (P + \beta Q)\beta(sR + rS) = \beta(sPR + rPS) + \beta^2(sQR + rQS) = \beta rs(P + \beta Q) = \beta rs F_1$$

It can be seen that each even power of F_1 is a scalar multiple of the matrix F_1^2, which we denote by

$$B = F_1^2 = \beta(sR + rS) \qquad [A.6]$$

while each odd power is a scalar multiple of F_1 itself, since

$$BF_1 = F_1 B = \beta rs F_1 \qquad [A.7]$$

Thus

$$F_1^2 = B, \qquad F_1^3 = \beta rs F_1$$
$$F_1^4 = B^2 = \beta rs B, \quad F_1^5 = (\beta rs)^2 F_1, \text{ and so forth} \qquad [A.8]$$

In general,

$$F_1^{2k+1} = (\beta rs)^k F_1$$
$$F_1^{2k+2} = (\beta rs)^k B \qquad [A.9]$$

Finally,

$$e^{\alpha F_1} = I + \sum_{k=0}^{\infty} \frac{(\alpha F_1)^{2k+1}}{(2k+1)!} + \sum_{k=0}^{\infty} \frac{(\alpha F_1)^{2k+2}}{(2k+2)!}$$
$$= I + \frac{F_1}{\sqrt{\beta rs}} \sum_{k=0}^{\infty} \frac{(\alpha\sqrt{\beta rs})^{2k+1}}{(2k+1)!} + \frac{B}{\beta rs} \sum_{k=0}^{\infty} \frac{(\alpha\sqrt{\beta rs})^{2k+2}}{(2k+2)!}$$
$$= I + \frac{F_1}{\sqrt{\beta rs}} \sinh(\alpha\sqrt{\beta rs}) + \frac{F_1^2}{\beta rs} [\cosh(\alpha\sqrt{\beta rs}) - 1] \qquad [A.10]$$

As a simple example, we reconsider the case of $(110)[\bar{1}12]$ compression, for which $e^{\alpha F_1}$ has already been evaluated in Eq. [33]. For this case, we have $\beta = 1$, $r = s = (1/3)\sqrt{6} = 2/\sqrt{6}$, and F_1 is given by Eq. [31]. Hence

$$\frac{F_1}{\sqrt{\beta rs}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 1 \end{bmatrix}, \quad \frac{F_1^2}{\beta rs} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 1 \end{bmatrix}$$

Substitution into Eq. [A.10] then gives, with $\varphi = 2a/\sqrt{6}$,

$$e^{\alpha F_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 1 \end{bmatrix} \sinh\varphi + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 1 \end{bmatrix} (\cosh\varphi - 1)$$

$$= \begin{bmatrix} (\cosh\varphi - \sinh\varphi) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}}(\cosh\varphi + \sinh\varphi - 1) & (\cosh\varphi + \sinh\varphi) \end{bmatrix} = \begin{bmatrix} e^{-\varphi} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{\sqrt{2}}(1 - e^{\varphi}) & e^{\varphi} \end{bmatrix}$$

in agreement with Eq. [33].

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